GPCO 2022 – 8th German-Polish Conference on Optimization

Conference Program

September 27-30, 2022
Apolda, Germany

FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

DFG Deutsche Forschungsgemeinschaft

Conference Program
8th German-Polish Conference on Optimization
September 27-30, 2022, Apolda

	Tuesday	Wednesday	Thursday	Friday
09:00 - 09:50		Maciej Smołka	Nikolai Osmolovskii	Gerd Wachsmuth
09:50 - 10:20	Plenary Talk Contributed Talk Coffee Break	Thomas Jahn	Benedikt Ohse	Frank Fischer
10:20 - 11:00		☕	☕	☕
11:00 - 11:30		Tetyana Romanova	Joachim Gwinner	Jonas Marko
11:30 - 12:00		Daniel Hoff	Patrick Mehlitz	Daniel Dörfler
12:00 - 12:30		Axel Kröner	Mina Saee	Zouhour Jaouadi
12:30 - 14:00	Lunch Break	Lunch Break	Lunch Break	Lunch Break
14:00 - 14:50	Agnieszka Malinowska	Excursion	Anja Fischer	
14:50 - 15:20	John Martinovic		Monika Syga	
15:20 - 16:00	☕		☕	
16:00 - 16:30	Andreas Fischer		Armin Fügenschuh	
16:30 - 17:00	Giovanni Bruccola			
17:00 - 17:30	Jerzy Grzybowski		Krzysztof Rutkowski	
17:30 - 18:00	Mario Jelitte		The Hung Tran	

Optimal control of the fractional Cucker–Smale model

Agnieszka B. Malinowska
a.malinowska@pb.edu.pl
Bialystok University of Technology, Poland

During the talk, I will present results concerning a sparse flocking control for the fractional Cucker–Smale multi-agent model. The Caputo fractional derivative, in the equations describing the dynamics of a consensus parameter, makes it possible to take into account in the self-organization of group its history and memory dependency. External control is designed based on necessary conditions for a local solution to the appropriate optimal control problem. Numerical simulations demonstrate the effectiveness of the control scheme.
The talk is based on the following papers [1, 2].

References

[1] Almeida Ricardo, Kamocki Rafał, Malinowska Agnieszka B., Odzijewicz Tatiana, On the necessary optimality conditions for the fractional Cucker–Smale optimal control problem, *Communications in Nonlinear Science and Numerical Simulation*, 96, (2021), 1–22.

[2] Almeida Ricardo, Kamocki Rafał, Malinowska Agnieszka B., Odzijewicz Tatiana, On the existence of optimal consensus control for the fractional Cucker–Smale model, *Archives of Control Sciences*, 30(4), (2020), 625–651.

A Combinatorial Flow-based Formulation for Temporal Bin Packing Problems

John Martinovic
Nico Strasdat
José Valério de Carvalho
Fabio Furini

We consider two neighboring generalizations of the classical bin packing problem: the temporal bin packing problem (TBPP) and the temporal bin packing problem with fire-ups (TBPP-FU). In both cases, the task is to arrange a set of given jobs, characterized by a resource consumption and an activity window, on homogeneous servers of limited capacity. To keep the operational costs but also the energy consumption low, TBPP is concerned with minimizing the number of servers in use, whereas TBPP-FU additionally takes into account the switch-on processes required for their operation. In both cases, challenging integer optimization problems are obtained, which can differ significantly from each other despite the seemingly only marginal variation of the problems. In the literature, a branch-and-price method enriched with many preprocessing steps (for the TBPP) and compact formulations (for the TBPP-FU), benefiting from numerous reduction methods, have emerged as, currently, the most promising solution methods. In this talk, we introduce, in a sense, a unified solution approach for both problems based on graph theory. Any scientific contributions in this direction failed so far because of the exponential size of the associated networks. The approach we present here does not change the theoretical exponentiality itself, but it can make it controllable by clever construction of the resulting graphs. In particular, this leads to the fact that for the first time all classical benchmark instances (and even larger ones) for the two problems can be solved – in times that significantly improve those of the previous approaches.

An Extended Local Convergence Theory for Newton-type Methods Applied to Complementarity Systems

Andreas Fischer
andreas.fischer@tu-dresden.de
Technische Universität Dresden, Institute of Numerical Mathematics, Germany

Nico Strasdat
Technische Universität Dresden, Institute of Numerical Mathematics, Germany

Some classes of nonlinear complementarity systems, like optimality conditions for generalized Nash equilibrium problems, typically have nonisolated solutions. Their reformulation as a constrained or unconstrained system of equations is often done by means of a nonsmooth complementarity function. Degenerate solutions then lead to points where the reformulated system is nonsmooth. Newton-type methods can have difficulties close to solutions that are nonisolated and degenerate. For this case, it is known that the LP-Newton method or a constrained Levenberg–Marquardt method may show local superlinear convergence provided that the complementarity function is piecewise linear [2]. These results rely on error bounds for active pieces of the reformulation [3]. On the basis of a somewhat different Index Error Bound Condition, we show that a related result can be obtained for a complementarity function that is not piecewise linear. To this end, a new convergence framework is developed that allows significantly larger steps. Then, by means of a a sophisticated analysis of the constrained Levenberg–Marquardt method, the regularization parameter can be chosen so that local superlinear convergence to a (possibly degenerate and nonisolated) solution with an R-order of at least 4/3 is achieved. Moreover, several known local convergence results can be recovered, like those in [1, 2, 3].

References

[1] R. Behling, A. Fischer, K. Schönefeld, N. Strasdat, A special complementarity function revisited. *Optimization* **68** (2019) 65–79

[2] F. Facchinei, A. Fischer, M. Herrich, A family of Newton methods for nonsmooth constrained systems with nonisolated solutions. *Mathematical Methods of Operations Research* **77** (2013) 433–443

[3] A. Fischer, M. Herrich, A.F. Izmailov, M.V. Solodov, Convergence conditions for Newton-type methods applied to complementarity systems with nonisolated solutions. *Computational Optimization and Applications* **63** (2016) 425–459

Finding global solutions for a class of possibly nonconvex QCQP problems through the S-lemma

Giovanni Bruccola
giovanni.bruccola@ibspan.waw.pl
Systems Research Institute, Polish Academy of Science

In this talk we provide necessary and sufficient (KKT) conditions for global optimality for a new class of possibly nonconvex quadratically constrained quadratic programming (QCQP) problems, denoted by S-QCQP. The proof relies on a generalized version of the S-Lemma, stated in the context of general QCQP problems. Moreover, we prove the exactness of the SDP and the SOCP relaxations for S-QCQP.

Based on the join work with Ewa M. Bednarczuk
Systems Research Institute, Polish Academy of Sciences

Minimal pairs of convex sets which share a recession cone [1]

Jerzy Grzybowski
jgrz@amu.edu.pl
Ryszard Urbański
Adam Mickiewicz University, Poland

Robinson [3] introduced a quotient space of pairs of unbounded convex sets which share their recession cone. We investigate minimal pairs of unbounded convex sets, i.e. minimal representations of elements of Robinson's quotient spaces. We prove that a minimal pair having property of translation is reduced. In the case of pairs of two-dimensional sets we give a formula for an equivalent minimal pair, present a criterion of minimality of a pair of sets and prove reducibility of all minimal pairs. We generalize Shephard–Weil–Schneider's criterion for polytopal summand of a compact convex set (Th. 3.2.11 in [4]) to unbounded convex sets. Finally, we apply minimal pairs of unbounded convex sets to finding Hartman's minimal representation of differences of convex functions [2].

References

[1] J. Grzybowski, R. Urbański, Minimal pairs of convex sets which share a recession cone. *SIAM Journal on Optimization* **32**(2022) (2), 1049–1068.

[2] Ph. Hartman, On functions representable as a difference of convex functions. *Pac. J. Math.* **9** (1959), 707–713.

[3] S. Robinson, An embedding theorem for unbounded convex sets. *Technical Summary Report No.* 1321, Madison Mathematics Research Center, University of Wisconsin (1973) 1–23.

[4] R. Schneider, Convex Bodies: The Brunn-Minkowski Theory. *Encyclopedia of Mathematics and its Applications*, **151**. Cambridge University Press, Cambridge 2014.

Constrained Lipschitzian Error Bounds and Noncritical Solutions of Constrained Equations

A. Fischer
Technische Universität Dresden, Germany

A. F. Izmailov
Lomonosov Moscow State University, Russia

M. Jelitte
mario.jelitte@tu-dresden.de
Technische Universität Dresden, Germany

For many years, local Lipschitzian error bounds for systems of equations have been successfully used for the design and analysis of Newton-type methods. There are characterizations of those error bounds by means of first-order derivatives like a recent result on critical solutions of nonlinear equations [3]. We aim at extending this result in two directions which shall enable, to some extent, to include additional constraints and to consider mappings with reduced smoothness requirements. This leads to new necessary as well as sufficient conditions for the existence of error bounds [2]. In combination with results from [1], we will formulate conditions, ensuring that a min-reformulation with slack-variables of a nonlinear complementarity problem provides a constrained Lipschitzian error bound.

References

[1] A. Fischer, M. Herrich, A.F. Izmailov, M.S. Solodov, Convergence conditions for Newton-type methods applied to complementarity systems with nonisolated solutions. *Comput. Optim. Appl.* **63**, 425–459 (2016)

[2] A. Fischer, A.F. Izmailov, M. Jelitte, Constrained Lipschitzian error bounds and noncritical solutions of constrained equations. *Set-Valued Var. Anal.* **29**, 745–765 (2021)

[3] A.F. Izmailov, A.S. Kurennoy, M.V. Solodov, Critical solutions of nonlinear equations: Stability issues. *Math. Program.* **168**, 475–507 (2018)

Solving difficult inverse problems using evolutionary algorithms

Maciej Smołka
smolka@agh.edu.pl
Institute of Computer Science, AGH University of Science and Technology, Poland

When solving inverse problems we face various difficulties. One important group of these obstacles is related to the ill-posedness. Namely, a considered problem can have a multitude of solutions for any input data. Those solutions can be isolated points in the parameter domain or, much worse, they can form sets with nonempty interior. Evolutionary algorithms are a group of methods repeatedly used to handle ill-posed inverse problems. They can potentially detect many solutions but simple strategies of this group typically fail to do so. This presentation shows some ways of using especially-designed evolutionary strategies accompanied with some auxiliary methods to overcome the above-mentioned obstacles and solve ill-posed inverse problems using moderate computational resources.

On the optimal constants in the two-sided Stechkin inequalities

Thomas Jahn
thomas.jahn@ku.de
Katholische Universität Eichstätt-Ingolstadt, Germany

Tino Ullrich
tino.ullrich@mathematik.tu-chemnitz.de
Faculty of Mathematics, Technische Universität Chemnitz, 09107 Chemnitz, Germany

The ℓ_1-norm of a monotonically decreasing sequence of nonnegative numbers can be sandwiched by a sum involving the best n-term approximation errors measured in the ℓ_q-norm, appropriately scaled by positive constants. But what are the optimal constants? In this talk, you will get in touch with the contributions of Copson, Stechkin, Pietsch, Temlyakov, and Bennett, the geometry behind the problem, and variants of the inequality, where the ℓ_1-norm is replaced by its weak counterpart or where sums are replaced by integrals.

References

[1] T. Jahn, T. Ullrich, On the optimal constants in the two-sided Stechkin inequalities. *J. Approx. Theory* **269**, Article no. 105607l, 2021.

Optimization of Sphere Packing Problems

Tetyana Romanova
tarom27@yahoo.com
Academy of Sciences of Ukraine
A. Pidhornyi Institute of Mechanical Engineering Problems, Kharkiv

Petro Stetsyuk
Academy of Sciences of Ukraine
V.M. Glushkov Institute of Cybernetics, Kyiv

Optimization problems for packing spheres in a bounded domain (container) with distance and balancing conditions are considered. Dense and sparse formulations were studied [1, 2]. The first problem aims at minimizing the container's volume, while the second maximizes the minimal Euclidean distance between each pair of spheres, as well as between each sphere and the boundary of the container. Mathematical models and solution algorithms for both problems are shown. By means of a penalty function approach, the first problem is transformed to an unconstrained minimization of a nonsmooth function. To find a good (local) solution of the problem, a modification of Shor's r-algorithm is used and combined with a multistart strategy for randomly chosen starting points. The sparse packing problem is formulated as a nonlinear optimization problem. The resulting large-scale problems are treated by the IPOPT solver combined with a decomposition algorithm. Some numerical results are presented for circular containers.

Both packing problems lead to large-scale optimization problems. To cope with this, special-purpose decomposition techniques are used to form a sequence of problems of lower dimensions. For some small instances global solutions were found for both formulations using the open-source global solver BARON [3].

References

[1] P. Stetsyuk, T. Romanova, G. Scheithauer, On the global minimum in a balanced circular packing problem. *Optimization Letters* **10** (2016) 1347–1360

[2] T. Romanova et al., Sparsest balanced packing of irregular 3D objects in a cylindrical container, *European Journal of Operational Research* **291** (2021) 84–100

[3] N.V. Sahinidis, BARON 21.1.13: Global Optimization of Mixed-Integer Nonlinear Programs, User's manual, 2021

Lower-level value function formulation for optimistic semivectorial bilevel optimization

Daniel Hoff
daniel.hoff@tu-ilmenau.de
Technische Universität Ilmenau, Germany

In bilevel optimization one considers an optimization problem, the upper-level problem, which contains the solution set of another optimization problem, the lower-level problem, as a constraint. Within this talk, we assume that the objective function on the lower-level is vector-valued. Moreover, we follow the optimistic approach, i.e., we choose the weakly efficient points of the lower-level problem that are most suitable for the upper-level objective function. Thereby, we assume, among other, that the objective and constraint functions are sufficiently smooth and a regularity condition like the MFCQ holds at all feasible points for the lower-level problem. This allows us to ensure the upper semicontinuity of the lower-level weakly efficient set map, which is defined by the multiobjective parametric optimization problem on the lower-level. Furthermore, this assumption allows us to derive existence results for minimal and ε-minimal points for optimistic semivectorial bilevel problems. As a next step, we follow the approach of replacing the weakly efficient set of the lower-level problem by using the optimal value function and obtain the so-called lower-level value function reformulation. This provides a starting point for bounding the optimal value of the semivectorial bilevel problem. We reach this aim by bounding the value function of the lower-level by appropriate lower and upper bound sets. This provides a base for branch-and-bound methods in optimistic semivectorial bilevel optimization.

Boundary control of fluid-structure interactions of linear elasticity with Navier-Stokes equations with mixed-boundary conditions in a channel

Axel Kröner
axel.kroener@mathematik.uni-halle.de
Martin-Luther-Universität Halle-Wittenberg, Germany

In this talk we consider a fluid-structure interaction problem given by the steady Navier Stokes equations coupled with linear elasticity taken from [1]. An elastic body surrounded by a liquid in a rectangular domain is deformed by the flow which can be controlled by the Dirichlet boundary condition at the inlet. On the walls along the channel homogeneous Dirichlet boundary conditions and on the outflow boundary do-nothing conditions are prescribed. We recall existence results for the nonlinear system from that reference and to formulate necessary optimality conditions we analyze the control to state mapping generalizing the results of [2] to the setting of the nonlinear Navier-Stokes equation for the fluid and the situation of mixed boundary conditions in a domain with corners. This is joint work with M. Hintermüller (WIAS).

References

[1] I. Lasiecka, K. Szulc, and A.Zochoswki, Boundary control of small solutions to fluidstructure interactions arising in coupling of elasticity with Navier-Stokes equation under mixed boundary conditions. *Nonl. Anal.: Real World Appl.*, **44**, 2018, pp. 54–85.

[2] T. Wick, W. Wollner, On the differentiability of fluid-structure interaction problems with respect to the problem data. *J. Math. Fluid Mech.*, **21**(3), 2019.

Second-Order Sufficient Optimality Conditions and Strong sub-Regularity of the Optimality Mapping in Extremal Problems with Constraints

Nikolai P. Osmolovskii
osmolov@ibspan.waw.pl
Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland

Perturbations in three types of nonlinear extremal problems with constraints are considered: the problem of mathematical programming in a Banach space, the general problem of the calculus of variations, and the Bolza-type optimal control problem with a control constraint $u(t) \in U$, where U is given by a finite number of smooth inequalities $G_i(u) \leq 0$ with linearly independent gradients of active constraints. In each problem, two norms are considered: the strong norm (L^∞) and the weak norm (L^2) – for perturbations ω of the problem's constraints, as well as for the principal variables x and for the dual variables λ. The notion of "strong metric subregularity" (SMsR) of the optimality mapping, which is related to the first-order optimality extremum conditions, is discussed. The SMsR property ensures that if the perturbations ω of the constraints and the corresponding variations Δx of the basic variables are sufficiently small in strong norms, then there exists a constant $\kappa > 0$ such that the weak norms of the variations Δx and $\Delta\lambda$ of the principal and dual variables are linearly estimated using the weak norm of constraint perturbations: $\|\Delta x\| \leq \kappa\|\omega\|$ and $\|\Delta\lambda\| \leq \kappa\|\omega\|$. We also formulate a second-order sufficient condition for a local minimum in each problem [1]. The main result recently obtained jointly with Vladimir Veliov [2] for each problem is that the second-order sufficient condition for a local minimum is also a sufficient condition for the SMsR property.

References

[1] Nikolai P. Osmolovskii, Second-order sufficient optimality conditions for control problems with linearly independent gradients of control constraints. *ESAIM: Control, Optimisation and Calculus of Variations* **Vol. 18** (Issue 2, April 2012), pp. 452–482

[2] Nikolai P.Osmolovskii, Vladimir M. Veliov. On the strong subregularity of the optimality mapping in mathematical programming and calculus of variations. *Journal of Mathematical Analysis and Applications* **Vol. 500** (Issue 1, 1 August 2021), 125077

Performance Space Supported Design of Analog Electronic Circuits

Benedikt Ohse
benedikt.ohse@eah-jena.de
Ernst-Abbe-Hochschule Jena, Germany

Christopher Schneider
Ernst-Abbe-Hochschule Jena, Germany

Performance spaces as representations for attainable image spaces of analog electronic circuits are well-suited for supporting their conventional design process. The design task can be formulated as non-convex multi-objective optimization problem. We apply a numerical approach based on the normal-boundary intersection method to model two example circuits. Furthermore, the impact of a power limitation and relaxation of design constraints on the resulting performance spaces is evaluated. Thus, we show how these models are supporting conclusions for important design decisions. We also present a concept for visualizing active constraints on the performance space boundary to directly identify responsible circuit parts. Another topic will be the method itself and especially its parallelization.

An Invitation to Uncertainty Qualification in Variational Inequalities

Joachim Gwinner
joachim.gwinner@unibw-muenchen.de
Universität der Bundeswehr München, Germany

The talk is based on the recent book [1]. We give an overview on random/stochastic variational inequalities ranging from theory to numerical solution methods and to applications in operations research.

References

[1] J. Gwinner, B. Jadamba, A.A. Khan, F. Raciti, *Uncertainty Quantification in Variational Inequalities. Theory, Numerics, and Applications*, Boca Raton, FL: CRC Press; xviii, 386 p. (2022).

On the directional asymptotic approach in optimization theory

Patrick Mehlitz
mehlitz@b-tu.de
BTU Cottbus–Senftenberg & University of Mannheim, Germany

In this talk, it will be demonstrated that for a fixed order $\gamma \geq 1$, each local minimizer of a rather general nonsmooth optimization problem in Euclidean spaces is either Mordukhovich-stationary in the classical sense (corresponding to stationarity of order 1), satisfies stationarity conditions in terms of a coderivative construction of order γ, or is approximately stationary with respect to a critical direction as well as γ in a certain sense. By ruling out the latter case with a constraint qualification not stronger than directional metric subregularity, we end up with new necessary optimality conditions comprising a mixture of limiting variational tools of order 1 and γ. These abstract findings are carved out for the broad class of geometric constraints. The talk closes by illustrating these results in the context of standard nonlinear and nonlinear semidefinite programming.

This talk is based on joint work with Matúš Benko, University of Vienna.

A Bundle Newton Method for Min-Max Problem

Mina Saee
m.saee@math.uni-goettingen.de
Georg-August-Universität Göttingen, Germany

Prof. D. Russell Luke
Georg-August-Universität Göttingen, Germany

Prof. Adrian S. Lewis
Cornell University, USA

We present a Bundle Newton method for minimizing the maximum of smooth convex functions which incorporates recent developments in nonsmooth analysis. Our proposed method is a two-phase approach where in the first phase random sampling and ”cutting quadratics” are applied to identify the active manifolds at the solution. This phase of the algorithm is finitely terminating. The second phase of the algorithm proceeds along the lines of conventional approaches, though our analysis shows superlinear convergence without the usual assumptions of strong convexity

References

[1] J. V. Burke, A. S. Lewis, and M. L. Overton. Approximating subdifferentials by random sampling of gradients. *Mathematics of Operations Research*, **Vol.** 29, pp. 567-584, 2002.

[2] J. B. Hiriart-Urruty, C. Lemarechal. Convex Analysis and Minimization Algorithms, I & II, . *Grundlehren Math, Wiss. Springer-Verlag*, New York,**Vol.** 305-306, 1993.

[3] M. Fukushima, Z.Q. Luo, and P. Tseng. A sequential quadratically constrained quadratic programming method for differentiable convex minimization. *SIAM Journal on Optimization*, **Vol.**13, pp. 1098-1119, 2003.

[4] Y. E. Nesterov, A. S. Nemirovskii. Interior-Point Polynomial Algorithms in Convex Programming. *SIAM Publications*, Philadelphia, 1994.

[5] R. T. Rockafellar. Convex Analysis. *Princeton University Press*, 1970.

[6] R. T. Rockafellar, R. J. Wets. Variational Analysis. *Grundlehren Math, Wiss. Springer-Verlag.* , Berlin, 3 edition, 2009.

[7] M. V. Solodov. On the sequential quadratically constrained quadratic programming methods. *Mathematics of Operations Research.*, **Vol**. 29, pp. 64-79, 2004.

[8] J. F. Sturm. Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones. *Optimization Methods and Software*, **Vol.** 11-12, pp. 625-653, 1999.

Non-linear combinatorial optimization meets facility layout problems

Anja Fischer
anja2.fischer@tu-dortmund.de
TU Dortmund University, Management Science, Germany

Given a set of departments and the pairwise transport weights between them the facility layout problem (FLP) looks for a non-overlapping arrangement of the departments such that the weighted sum of the distances between the departments is minimized. Several variants of the FLP are known in the literature where the structure of the path system, which allows moving between the departments, is restricted. Important special cases are the Single-Row FLP (SRFLP) and the Double-Row FLP (DRFLP) which contain only one path. Examples with more paths contain the Multi-Bay FLP or layouts with pier-type material flow patterns. The best (mixed-) integer linear programming approach for the SRFLP uses betweenness variables [1] which correspond to products of ordering variables. In this talk we will give an overview of current results on exact solution approaches for specific FLPs. We present new results on the betweenness polytope which allows solving large SRFLP instances. Apart from this we show how the betweenness model is used in enumerative approaches for solving FLPs with a more complicated path structure.

Parts of this talk are based on joint work with Frank Fischer, Kerstin Maier, Mirko Dahlbeck and Philipp Hungerländer.

References

[1] A. R. S. Amaral, A new lower bound for the single row facility layout problem. *Discrete Applied Mathematics* **Vol.** 157(1), pp. 183–190.

The generalized S-lemma

Monika Syga
m.syga@mini.pw.edu.pl
Warsaw Univesity of Technology, Poland

The S-lemma is well known result, which has many applications i.e in quadratic optimization, control theory and linear algebra. In this talk we present a generalized version of the S-lemma for any two functions defined on a nonempty set X. In our approach we do not exploit the linear or topological properties of the set X.

Mixed-Integer Programming with Differential Equations Constraints

Armin Fügenschuh
fuegenschuh@b-tu.de
Brandenburg University of Technology, Germany

An area of research that recently gained momentum are Mixed-Integer Partial Differential Equation Constrained Optimization (MIPDECO) problems, which refers to the inclusion of continuous physical phenomena described by ODEs or PDEs into discrete decision processes. For instance, Fügenschuh et al. [3, 2] describe network flow problems in which the transport equations are included, Koch et al. [7] and Hahn et al. [6] combine gas dynamics with discrete decisions related to the control of natural gas networks, and Frank et al. [1] consider the shortest path problem together with the heat equation. We will refer to this class of problems as Mixed-Integer PDE-Constrained Optimization (MIPDECO) problems, a term introduce in [8]; this notion includes ODEs as a one-dimensional special case of PDEs. A possible way for solving MIPDECO problems is to adapt known approximation techniques for differential equations (such as finite-difference methods), in order to obtain a classical Mixed-Integer Linear Program with only finitely many constraints and variables. In the case of linear PDEs, it is sometimes possible to transfer the solution of the PDE into a preprocessing step. That is, it can be efficient to solve the PDE for a basis of the control space and apply the principle of superposition to obtain solutions by linear combinations of the basis. In addition to that, splitting the solution process of the PDE and the MILP has the additional benefit that other solution methods, such as Galerkin methods (finite element methods) can be applied more readily. We demonstrate these ideas for some test problems, such as mission and trajectory planning for unmanned aerial vehicles [4], the firefighter deployment in a wildfire incident [5], a contaminated subsurface water flow controlled by filtration stations [5], and additive manufacturing (3D-printing) [9].

References

[1] M. Frank, A. Fügenschuh, M. Herty, and L. Schewe. The Coolest Path Problem. *Networks and Heterogeneous Media*, 5(1):143–162, 2010. DOI: 10.3934/nhm.2010.5.143.

[2] A. Fügenschuh, S. Göttlich, C. Kirchner, M. Herty, and A. Martin. Efficient Reformulation and Solution of a Nonlinear PDE-Controlled Flow Network Model. *Computing*, 85(3):245–265, 2009. DOI: 10.1007/s00607-009-0038-7.

[3] A. Fügenschuh, M. Herty, A. Klar, and A. Martin. Combinatorial and Continuous Models and Optimization for Traffic Flow on Networks. *SIAM Journal on Optimization*, 16(4):1155–1176, 2006. DOI: 10.1137/040605503.

[4] A. Fügenschuh, D. Müllenstedt, and J. Schmidt. Flight Planning for Unmanned Aerial Vehicles. *Military Operations Research*, 26(3):49–71, 2021. URL: https://www.jstor.org/stable/27070890.

[5] F. Gnegel, A. Fügenschuh, M. Hagel, S. Leyffer, and M. Stiemer. A Solution Framework for Linear PDE-Constrained Mixed-Integer Problems. *Mathematical Programming*, 188:695–728, 2021. DOI: 10.1007/s10107-021-01626-1.

[6] M. Hahn, S. Leyffer, and V. M. Zavala. Mixed-Integer PDE-Constrained Optimal Control of Gas Networks. *Technical report, Argonne National Laboratory ANL/MCS-P7095-0817*, 2017. Available at URL: https://wiki.mcs.anl.gov/leyffer/images/2/27/GasNetMIP.pdf (visited on 05/15/2022).

[7] T. Koch, B. Hiller, M. E. Pfetsch, and L. Schewe. Evaluating gas network capacities. *Society for Industrial and Applied Mathematics (SIAM)*, Philadelphia, 2015. DOI: 10.1137/1.9781611973693.

[8] S. Leyffer. Mixed-Integer PDE-Constrained Optimization. In A. Wiegele L. Liberti, S. Sager, editor, *Mixed-integer Nonlinear Optimization: A Hatchery for Modern Mathematics*, volume 46/2015, pages 2738–2740. Mathematisches Forschungsinstitut Oberwolfach, 2015. DOI: 10.4171/OWR/2015/46.

[9] J. Schmidt, M. Bähr, G. Radow, J. Buhl, M. Bambach, M. Breuß, and A. Fügenschuh. Stable Honeycomb Structures and Temperature Based Trajectory Optimization for Wire-Arc Additive Manufacturing. *Optimization and Engineering*, 22:913–974, 2021. DOI: 10.1007/s11081-020-09552-5.

Abadie Condition under Relaxed Constant Rank Qualification+

Krzysztof Rutkowski
k.rutkowski@uksw.edu.pl
Cardinal Stefan Wyszyński University Warsaw, Poland

In this presentation we consider infinite programming problems with constraint sets defined by systems of infinite number of inequalities and equations given by continuously differentiable functions defined on Banach spaces, namely we consider the problem

$$\begin{aligned} &\text{Minimize}_{x\in\mathcal{F}}\, f_0(x) \\ \mathcal{F} = &\left\{ x \in E \;\middle|\; \begin{array}{ll} g_i(x) = 0, & i \in I_0, \\ g_i(x) \le 0, & i \in I_1 \end{array} \right\}, \end{aligned} \qquad (P_0)$$

where E is a Banach space, $I_0 \cup I_1$ is countable, $g_i : E \to \mathbb{R}$ are of class C^1 and $\sum_{i\in I_0\cup I_1} g_i(x)^2 < +\infty$ for all $x \in U$, $U \subset E$ - open.

In this approach we represent such systems of constraints with the help of coefficients in a given Schauder basis. We provide sufficient conditions under which Abadie condition holds and we discuss the existence of Lagrange multipliers.

The main tools are: Relaxed Constant Rank Qualification+, Rank Theorem and Ljusternik Theorem. The talk is based on the common work [1].

References

[1] E. M. Bednarczuk, K. W. Leśniewski, K. E. Rutkowski; Abadie condition for infinite programming problems under Relaxed Constant Rank Constraint Qualification Plus, *arxiv: https://doi.org/10.48550/arXiv.2112.07460*, 2022

Duality for Composite Optimization Problem within the Framework of Abstract Convexity

The Hung Tran
tthung@ibspan.waw.pl
System Research Institute-Polish Academy of Science, Warsaw, Poland

Ewa Bednarczuk
System Research Institute-Polish Academy of Science, Warsaw, Poland

We study conjugate and Lagrange dualities for composite optimization problems within the framework of abstract convexity. We provide conditions for zero duality gap in conjugate duality. For Lagrange duality, intersection property is applied to obtain zero duality gap. Connection between Lagrange dual and conjugate dual is also established. Examples related to convex and weakly convex functions are given.

Newton's problem of least resistance

Gerd Wachsmuth
gerd.wachsmuth@b-tu.de
BTU Cottbus-Senftenberg, Germany

Newton's problem of least resistance asks for the shape of a convex body with a given base and a given height such that its resistance is minimal when travelling through a raw media. This problem can be modeled as follows. We denote by $\Omega \subset \mathbb{R}^2$ the given base, typically Ω is the unit ball. Now, the body is

$$P = \{(x, z) \in \Omega \times \mathbb{R} \mid u(x) \leq z \leq L\},$$

where $u \colon \Omega \to [0, L]$ is convex. When travelling in the negative z-direction, a simple modelling results in the resistance

$$\int_\Omega \frac{1}{1 + \|\nabla u(x)\|^2} \, \mathrm{d}x.$$

This expression has to be minimized w.r.t. $u \colon \Omega \to [0, L]$ being convex.

In the talk, we review some of the history, state some analytical results concerning the optimal shapes and address the numerical solution of the problem.

An asynchronous proximal bundle method

Frank Fischer
frank.fischer@uni-mainz.de
Johannes Gutenberg-University Mainz, Computer Science, Germany

Proximal bundle methods are well-known algorithms to solve non-smooth convex optimization problems. The basic idea is to iteratively compute new candidate points by solving a cutting plane model of the function and to evaluate the function at a new point. If the progress in the function value is good enough, the algorithm goes to the candidate point, otherwise the cutting plane model is improved.

We extend the classic iterative method into a fully asynchronous algorithm. This means that all function evaluations as well as solving the cutting plane model is done in parallel and may take arbitrary time. The results of all processes arrive over time and are combined to achieve global progress. This approach has several advantages over the classic approach, e. g. multiple different cutting plane models may be used to compute several candidates and multiple processes may evaluate the same functions at different candidate points at the same time.

We show that the algorithm converges under mild assumptions and can be used as a drop-in replacement for the classic sequential approach, i. e. no additional information besides a first-order oracle for the function evaluation is required. In particular, we show how the algorithm learns important structural properties of the functions to control the inaccuracy induced by the asynchronicity automatically such that overall convergence can be guaranteed.

On Integer Optimal Control Problems with Total Variation Regularization

Jonas Marko
markojo1@b-tu.de
BTU Cottbus-Senftenberg, Germany

Gerd Wachsmuth
BTU Cottbus-Senftenberg, Germany

We investigate the integer optimal control problem

$$\begin{aligned} &\text{Minimize} \quad F(u) + \beta \mathrm{TV}(u) \\ &\text{such that} \quad u(t) \in \{\nu_1, \ldots, \nu_d\} \subset \mathbb{Z} \text{ for a.a. } t \in (0, T), \end{aligned} \tag{P}$$

with $\beta > 0$, where $\mathrm{TV}(u)$ denotes the total variation of u, which penalizes jumps of the control. The contribution F is assumed to be differentiable, e.g. it could realize the tracking of the state given by an ODE dependent on u. The presence of the TV-term can be used to prove existence of a minimizer. Moreover, it averts rapid switching between multiple control levels in a short amount of time, which is desirable from an application point of view.

We show local optimality conditions of first and second order via a finite-dimensional switching point problem. Additionally, a non-local optimality condition treating back-and-forth switches will be formulated.

For a numerical solution, we propose a proximal gradient method. The emerging discretized subproblems will be solved by an algorithm which is polynomial in the mesh size and in the admissible control levels. An adaption of this algorithm can be used to handle subproblems of the trust-region method proposed in [1]. We show properties of the proximal gradient method and of the generated sequence of iterates. Finally, we showcase numerical results exemplary on two control problems, one governed by the Lotka-Volterra equations while the other is given as in [1, Chapter 5].

References

[1] Sven Leyffer, Paul Manns (2021) *Sequential Linear Integer Programming for Integer Optimal Control with Total Variation Regularization*, https://arxiv.org/abs/2106.13453.

On polyhedral approximation of unbounded convex sets with an application to convex vector optimization

Daniel Dörfler
daniel.doerfler@uni-jena.de
Friedrich Schiller University Jena, Germany

There are various solution concepts for vector optimization problems in the literature. Given a convex vector optimization problem (CVOP) it is in general only possible to compute an approximation of a solution. One of these approximate solution concepts considers a finite set of weakly efficient points that generate a polyhedral approximation to the upper image, a convex set associated with a given CVOP, in the sense that the Hausdorff distance between the two sets is at most a predefined value. It has recently been established, that approximate solutions to a CVOP exist if and only if it satisfies a condition known as self-boundedness.
We investigate the problem of approximating an unbounded convex set by a polyhedron from a geometric perspective and conclude that the Hausdorff distance is inadequate to quantify an approximation error. We further introduce a notion of polyhedral approximation that is dependent on two parameters and takes the recession cones of the involved sets into account. We show that polyhedral approximations under this notion are suitable to approximate line-free convex sets in the sense that a sequence of polyhedral approximations converges to the set as the parameters diminish. We apply this notion to propose an approximate solution concept for CVOPs.

Improving the Aerodynamic Performance of the Trans-Tokyo Bay Bridge using Reliability-Based Design Optimization

Zouhour Jaouadi
zouhour.jaouadi@uni-weimar.de
Bauhaus-Universität Weimar, Germany

Tajammal Abbas
WSP UK Limited, United Kingdom

Guido Morgenthal
Bauhaus-Universität Weimar, Germany

Tom Lahmer
Bauhaus-Universität Weimar, Germany

Slender structures such as bridges are prone to wind excitations. Therefore, special requirements from early stages of design have to be considered. Bridge decks, including any other element of the bridge, may experience Vortex-Induced Vibration (VIV) which is the result of wind-structure interaction. VIV occurs for a certain band of wind speeds, usually lower than the aeroelastic instability limit, and it has to be avoided since it can cause fatigue damage accumulation on structural elements. Mitigating the VIV effects consists of avoiding the coincidence of the vortex shedding frequency with the natural frequency of the structure, and delaying the VIV, which shifts the lock-in behavior to higher wind speed. Thus, approaches related to the modification of the geometrical boundary are considered, where the idea is based on diverting the oncoming flow by smoothing the leading and trailing edges. Within this framework, a study is evolved on the Trans-Tokyo Bay Bridge, which suffers from VIV effects. A new strategy based on Reliability-Based Design Optimization (RBDO) is proposed to find the optimal cross-sectional shape that ensures simultaneously low-cost design and mitigation of the VIV effects under uncertainty. Validation of the final design via Computational Fluid Dynamics (CFD) simulations is also presented.